TECHNICAL NOTE

APPLICABILITY OF CLOHESSY-WILTSHIRE MODEL FOR OPTIMAL LOW-THRUST RENDEZVOUS

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Abstract

This paper investigates the applicability of the continuous thrust Clohessy-Wiltshire model for the optimal low-thrust rendezvous in the orbit. While simplifying the non-linear equations of relative orbital motion, certain assumptions were taken in account by Clohessy and Wiltshire to linearize the system of equations. When the same model is used for optimization of the trajectory of a relative transfer in the orbit using low-thrust, because of the assumptions in the dynamic model there are errors in the result which in turn depends upon the initial conditions for the optimal control problem. A study has been done to investigate the possible trend of error in the control input by varying the initial conditions for the relative transfer and comparing the results with those obtained using non-linear equations. Further, the trend of error in control input has also been studied by increasing the altitude of the target spacecraft orbit. Optimization of the relative motion trajectory has been performed to account for minimum propellant expenditure.

Keywords: *Continuous Low-Thrust Transfer, Clohessy-Wiltshire Model, Minimum Propellant Control, Trajectory Optimization*

Nomenclature

μ = Standard gravitational parameter for Earth, 398600.4 km^3/s^2
φ = True anomaly, rad
Λ = Co-state vector, rad
U = Control input vector
A = Dynamics matrix
B = Control matrix
J = Performance index
n = Orbital frequency
R = Weight matrix
r = Relative distance between target and maneuvering spacecraft, km
r_t = Radius of target spacecraft orbit, km
t = time, s
u_x = Control input along x-axis, km/s^2
u_y = Control input along y-axis, km/s^2
u_z = Control input along z-axis, km/s^2
υ = Velocity of target spacecraft relative to maneuvering spacecraft, km/s
υ_x = Relative velocity of target spacecraft along x-axis, km/s
υ_y = Relative velocity of target spacecraft along y-axis, km/s
υ_z = Relative velocity of target spacecraft along z-axis, km/s
x = Component of relative distance along radius vector of target
y = Component of relative distance tangential to the velocity vector of target
z = Component of relative distance along the angular momentum of target orbit

Introduction

Orbital maneuvers using continuous low-thrust is a recent and popular field of study. The sources of low-thrust are usually the unconventional propulsion techniques such as, the electric propulsion and the solar-sail systems. A number of low-thrust missions have been deployed in the recent years [11]. Since, the magnitude of

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the thrust obtained from such propulsion systems is relatively small as compared with the thermal rocket engines [2], there is utmost requirement of an optimal control that should make the mission possible within the bounds of available thrust. For continuous low-thrust rendezvous, the objective of optimal control problem is to guide the maneuvering spacecraft from an initial state to a final state such that, the final position and velocity of the maneuvering spacecraft relative to the target spacecraft is zero, while minimizing the propellant consumption. On the basis of whether there are constraints on the applied input, since, due to obvious reasons, the thrust generated by the low-thrust propulsion has limits on its magnitude, the optimization can be constrained or unconstrained.

Clohessy and Wiltshire [1] proposed the approximate linear model for relative orbital transfer and it is a popular choice for trajectory optimization as compared to the non-linear model because of its fast convergence rate as compared to non-linear model and since, computational power available on-board the spacecraft is limited. Marinescu [3] presents the minimum propellant optimal rendezvous of the two spacecraft without accounting for any constraints on the control input and considering the circular and the elliptical reference orbits, sequentially. For the optimization for the circular reference orbit, he made use of the approximate linearized equation of motion similar to Clohessy-Wiltshire model [1]. He demonstrated that the maximum absolute values of the required control input falls in the domain of the low-thrust propulsion [2].

Euler [4] studied the optimal low thrust coplanar rendezvous between the maneuvering and the target spacecraft using the approximate equations of motion. He compared the optimal trajectory solution obtained from the approximate equations with the optimal trajectory solution obtained using the non-linear exact equations obtained using the Newton-Raphson method.

However, unlike the present paper, the study was made for coplanar transfer only. Hinz [6] studied the optimal low-thrust transfer between neighboring coplanar circular orbits by taking thrust acceleration as constant. Also, since we are dealing with a continuous thrust propulsion device, it is obvious that while optimizing the control it has to be ensured that the value of the thrust required in within the reach of the propulsion mechanism. Guelman and Aleshin [5] investigated the bounded, low-thrust, fixed-time fuel optimal constrained terminal approach direction rendezvous using the relative linearized equations of motion in which they have developed a minimum-fuel two stage solution. The first stage of the solution, optimal transfer from the initial conditions to an intermediate point in the final line approach was implemented and in the second stage, an optimal thrust program guaranteeing directional approach was applied. Optimal rendezvous with fixed terminal approach is achieved by minimizing overall fuel consumption as a function of the intermediate point relative location and velocity in the final line approach.

Clohessy-Wilshire model [1] approximates the equation of motion assuming circular reference orbit. After [1], a number of modified models were proposed to capture the effects of real world. Tschauner and Hempel [6] presents the approximate model of the equations of relative motion, however, compared to [1], elliptical reference orbits can also be accounted in the model. Carter and Humi [9] further modified the Clohessy-Wiltshire model [1] to include quadratic drag.

Carter and Humi [13] investigates the fuel optimal rendezvous near a point in the general Keplerian orbit assuming bounded thrust and constant exhaust velocity. It is demonstrated in [3] that the maximum absolute values of the required control input falls in the domain of the low-thrust propulsion [2]. Using the Tschauner and Hempel model [6], Sengupta and Vadali [14] worked out the analytical solution for power-limited optimal minimum fuel rendezvous near an elliptic orbit. Using the same model, Sharma et al. [15] presents an approach to the design of near-optimal feedback control laws, for minimum-fuel rendezvous between satellites in elliptic orbits of arbitrary eccentricity. The rendezvous problem for the nonlinear differential gravity model is solved by the application of neighboring optimal feedback control methodology used in conjunction with a nominal trajectory, obtained by solving the related minimum-fuel feedback control problem for the linear Tschauner-Hempel equations [6], analytically. Shao et al. [16] studied the optimal fuel control design of the nonlinear spacecraft rendezvous system with the collision avoidance constraint, and formulated an optimal control problem with state-constraints by using exact penalty function method to transform the constrained optimization problem into a sequence of approximate unconstrained optimization problems. It was shown that the solution of these unconstrained problems converges to the solution of the original problem.

Epenoy [17] designed a numerical method for computing constrained fuel-optimal continuous-thrust rendezvous trajectories. By adapting the idea of using smoothed exact penalty functions to the optimal control context, it is shown that it is possible to solve this kind of state-constrained problem through Pontryagin's minimum principle by simply solving a sequence of unconstrained problems.

The prime focus of research in the field of optimal low thrust relative orbital transfer is on the accurate modelling and formulation of the optimal control problem and obtaining optimal control using the approximate model. However, the applicability of the approximated model by varying the initial conditions of the maneuvering spacecraft, such as, the initial relative velocity and the initial relative distance, has not been examined. This paper thoroughly investigates the applicability limit of the Clohessy-Wiltshire model [1] for the continuous low-thrust relative transfer by varying the initial conditions and comparing the results obtained with the non-linear model.

Mathematical Model

Consider the target spacecraft located in an orbit of radius r_t around Earth. Another spacecraft located in another arbitrary orbit is equipped with a continuous low-thrust propulsion mechanism and is known as the maneuvering spacecraft. The maneuvering spacecraft has to rendezvous with the target spacecraft using continuous low-thrust propulsion within a specified time, known as the mission time. To analyze the problem conveniently, a local-vertical-local-horizontal coordinate frame, Hill's frame, is fixed on the target spacecraft. The Hill's frame is designated as the *x*-axis directed along the radius vector of the target spacecraft, *y*-axis directed along the velocity vector of the target spacecraft and *z*-axis points out of the plane, hence, completes the setup. The Hill's frame is shown in Fig.1.

If $\boldsymbol{U}$ is the control input in the form of continuous specific thrust:

$$\boldsymbol{U} = \begin{bmatrix} u_x & u_y & u_z \end{bmatrix}^T \tag{1}$$

Assuming there is no perturbation in the orbit and the motion of the spacecraft is purely Keplerian, the non-linear equations of relative motion for the continuous thrust transfer are:

$$\ddot{x} = 2\,\dot{\varphi}\,\dot{y} + \ddot{\varphi}\,y + \dot{\varphi}^2\,x + \mu\left\{\frac{1}{r^2} - \frac{x + r_t}{\left((x + r_t)^2 + y^2 + z^2\right)^{\frac{3}{2}}}\right\} + u_x \tag{2}$$

$$\ddot{y} = -2\,\dot{\varphi}\,\dot{x} - \ddot{\varphi}\,x + \dot{\varphi}^2\,y + \mu\left\{-\frac{y}{\left((x + r_t)^2 + y^2 + z^2\right)^{\frac{3}{2}}}\right\} + u_y \tag{3}$$

$$\ddot{z} = \mu\left\{-\frac{z}{\left((x + r_t)^2 + y^2 + z^2\right)^{\frac{3}{2}}}\right\} + u_z \tag{4}$$

where, φ is the true anomaly and $\mu = 398600.4\ \text{km}^3/\text{s}^2$ is the standard gravitational parameter for Earth.

By employing assumptions that the orbit of the target is circular, and that the relative distance between the target and the maneuvering spacecraft is much small as compared to the radius of the target's orbit, one can easily obtain the Clohessy-Wiltshire (CW) Eqn.(1) for relative orbital motion. CW model for the continuous thrust transfer is:

$$\ddot{x} = 3\,n^2\,x + 2\,n\,\dot{y} + u_x \tag{5}$$

$$\ddot{y} = -2\,n\,\dot{x} + u_y \tag{6}$$

$$\ddot{z} = -n^2\,z + u_z \tag{7}$$

n is the orbital frequency of the target's orbit obtained as:

$$n = \sqrt{\frac{\mu}{r_t^3}} \tag{8}$$

The above model can be represented in state-space as:

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} + B\boldsymbol{U} \tag{9}$$

where,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3\,n^2 & 0 & 0 & 2n & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -n^2 & 0 \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$\boldsymbol{X}$ is the state vector:

$$X = [x \ \dot{x} \ y \ \dot{y} \ z \ \dot{z}]^T \tag{12}$$

Minimum Propellant Control

For minimizing the propellant, the continuous thrust transfer models discussed in previous section are subjected to the performance index:

$$J = \frac{1}{2}\int_{t_i}^{t_f} (\boldsymbol{U}^{\boldsymbol{T}} R \, \boldsymbol{U}) \, dt \tag{13}$$

where, t_i and t_f are the initial and final time at which the transfer occurs and R is the weight matrix, which in this case is I_{3x3}.

Optimization of the Low-Thrust CW Model

Considering the continuous-thrust CW model of Eq.(9), if the co-state vector is:

$$\boldsymbol{\Lambda} = [\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4 \ \lambda_5 \ \lambda_6]^T \tag{14}$$

then, the Hamiltonian function is obtained as:

$$H = \frac{1}{2}\boldsymbol{U}^{\boldsymbol{T}} R \, \boldsymbol{U} + \boldsymbol{\Lambda}^{\boldsymbol{T}} (A \, \boldsymbol{X} + B \, \boldsymbol{U}) \tag{15}$$

For stationarity, the condition must be satisfied:

$$\frac{\partial H}{\partial \boldsymbol{U}} = 0 \tag{16}$$

which gives,

$$u_x = -\lambda_2 \tag{17}$$

$$u_y = -\lambda_4 \tag{18}$$

$$u_z = -\lambda_6 \tag{19}$$

Hence, Eq.(15) can be re-written as:

$$H = -\frac{1}{2}(\lambda_2^2 + \lambda_4^2 + \lambda_6^2) + \boldsymbol{\Lambda}^{\boldsymbol{T}} (A \, \boldsymbol{X}) \tag{20}$$

Applying the necessary condition for optimality, we get:

$$\dot{\Lambda} = -\left(\frac{\partial H}{\partial \boldsymbol{X}}\right)^T = -A^T \Lambda \tag{21}$$

Hence, collectively, the state and co-state equations can be written as:

$$\begin{bmatrix} \dot{\boldsymbol{X}} \\ \dot{\boldsymbol{\Lambda}} \end{bmatrix} = \begin{bmatrix} A & -B \\ 0 & -A^T \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{\Lambda} \end{bmatrix} \tag{22}$$

Eq.(22) represents a set of 12 linear differential equations, in which, the state equations are separable from the co-state equations and, hence, can be solved separately.

If the initial values of the co-states are:

$$\Lambda(t_t) = \Lambda_0 \tag{23}$$

Then, the co-state vector at a time can be expressed as:

$$\Lambda(t) = e^{-A^T t} \Lambda_0 \tag{24}$$

The state-vector at a time t is obtained as:

$$\boldsymbol{X}(t) \ e^{At} \boldsymbol{X}(t_i) + \int_0^t e^{A(t-\tau)} B \, \boldsymbol{U}(\tau) \, d\tau \tag{25}$$

where, $\boldsymbol{X}(t_i)$ is the initial value of the state-vector. The solution to the state-vector is obtained by substituting Eq.(24) in Eq.(25) and then, solving for the initial value of the co-state vector by applying the boundary conditions. The solution to the control input vector, $\boldsymbol{U}$, is obtained by substituting the values of co-state vector to Eqs.(17) - (19). The problem, hence, formulated is a two-point boundary value problem (2PBVP).

Optimization of the Low-Thrust Non-Linear Model

Since, the continuous thrust non-linear model discussed in Eqs.(2)-(4) cannot be converted into the state-space form, it cannot be solved analytically by separating the state and the co-state vectors like the low-thrust CW model in subsection A. It is hence, needed to be solved numerically. There are various effective methods available for the numerical solution of a two-point boundary value problem, such as the shooting method [10] and the collocation methods [12]. In this study, the non-linear 2PBVP is solved using MATLAB inbuilt function bvp4c which uses a collocation method for obtaining the optimal solution.

Numerical Examples

For this study, the target spacecraft is considered to be the International Space Station, which is in an approximate circular orbit of radius 6778.14 km. The initial state vector for relative position and relative velocity of the maneuvering spacecraft is given as:

$$\boldsymbol{X}(t = t_i) = [10 \;\; 0 \;\; 10 \;\; 0 \;\; 10 \;\; 0\,]^T \tag{26}$$

It is required to rendezvous the maneuvering spacecraft to target spacecraft using continuous low-thrust in the specified time, $t = \left(\frac{\pi}{n}\right) = 2776.8\ s$, where n is the orbital frequency of the target's orbit. Hence, the final state vector should be:

$$\boldsymbol{X}(t = t_i) = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\,]^T \tag{27}$$

Figure 2 shows the trajectory of control input (specific thrust) as obtained using the continuous-thrust CW model along x, y and z-axis, respectively.

It can be noticed that the maximum magnitude of the resultant specific thrust is bounded up to 0.05 m/s^2 which is well within the reach of low-thrust propulsion. Hence, the optimization process justifies the use of continuous low-thrust propulsion.

On comparing the specific thrust obtained using the continuous thrust CW model with that obtained using non-linear model, a maximum error of 0.18% is found. The error in specific thrust input has been shown in Fig.3.

Figure 4 shows the trajectory of relative distances along the three axes obtained using the continuous-thrust CW model, and it is evident from the plot that the maneuvering spacecraft is indeed covering the proposed mission within the stipulated time-limit, i.e., the relative distances along x, y and z-axis are reducing to zero at the final time. When the same simulation has been carried out using the continuous thrust non-linear model, a maximum error of 0.17% which is about 10.7 m, is found in the relative distance. The error in relative distance with the advance of mission time is shown in Fig.5.

Figure 6 shows the trajectory of the relative velocities along the three axes obtained using the continuous thrust CW model, the maximum magnitude of the relative velocity of the maneuvering spacecraft being close to 14 m/s. The maximum error in the computing the relative velocity on comparison with the optimization carried out using continuous thrust non-linear model is 0.16% and the error in relative velocity with the advance of mission time is shown in Fig.7.

Effect of Change of Initial Conditions on Optimization

To study the effect of change in the initial conditions on the validity of the optimal transfer done using continuous-thrust CW model, the maximum error in the control input has been plotted for different initial conditions of relative distance and relative velocity.

Varying the Initial Relative Distance

Consider that the target is located in a circular orbit of radius 6778.14 km and the initial state-vector is given as:

$$X(t = t_i) = \left[x_i \;\; 0 \;\; x_i \;\; 0 \;\; x_i \;\; 0\right]^T \tag{28}$$

To study the effect of increasing the initial relative distance between the maneuvering and the target spacecraft on the accuracy of the optimal transfer done using the low-thrust CW model, the maximum absolute error in the control input is calculated by comparing the results of optimal transfer obtained using the low-thrust CW model with the low-thrust non-linear equations model. The results of the simulation are plotted for different values of initial relative distance, varying it from 0 to 200 km along every axis, i.e., an overall relative distance of 346.4 km. The results have been plotted for different mission timings and are shown in Fig.8.

It can be inferred from Fig.8 that with the increase in the initial relative separation between the target and maneuvering spacecraft, the error in the control input increases almost linearly. It is also deduced from the study that as the time of the transfer increases, the slope of the error curve increases. Table-1 summarizes the results of simulations obtained by varying the initial relative distance between the maneuvering and the target spacecraft

Table-1 : Effect of Changing Initial Relative Distance

$T\,(s)$	$r\,(0)$ (km) for which ΔU_{max} is			
	1%	2%	5%	10%
$\pi/4n$	267.2	555	1588	4129
$\pi/2n$	125.2	250.8	632.9	1294
π/n	97.8	200.2	541	1254
$2\pi/n$	14.2	28.2	70.02	139
$4\pi/n$	11.3	22.46	55.23	107.1

Table-2 : Effect of Changing Initial Relative Velocity

$T\,(s)$	$\upsilon\,(0)$ (km) for which ΔU_{max} is			
	1%	2%	5%	10%
$\pi/4n$	0.346	1.883	5.398	11.59
$\pi/2n$	0.179	0.704	2.166	4.602
π/n	0.06	0.221	0.723	1.57
$2\pi/n$	-	-	0.361	0.673

of the above discussed example for different mission timings. It can be seen that for a mission timing as low as $T=\left(\frac{\pi}{4n}\right)=694.2\,s$, the continuous-thrust CW model has less than 1% error in the applied input calculation for a relative distance of up to 267.2 km. However, whether the amount of continuous thrust required to complete the transfer in such short duration can be delivered by the low-thrust mechanism, is another case. Hence, the applicability of continuous-thrust CW model for continuous low-thrust transfer is limited by the maximum error tolerance allowed by the guidance system of the maneuvering spacecraft subjected to the value of thrust being produced.

Varying the Initial Relative Velocity

Similar to the above case, the effect of increasing the initial relative velocity of the maneuvering spacecraft on continuous low-thrust rendezvous has been studied for different mission timings and the results obtained using the continuous thrust CW model have been compared with those obtained using continuous thrust non-linear model. Considering the target spacecraft to be in a circular orbit of radius 6778.14 km and the initial state vector is given as:

$$X\,(t=t_i) = \left[50\ \ \upsilon_i\ \ 50\ \ \upsilon_i\ \ 50\ \ \upsilon_i\right]^T \tag{29}$$

To study the effect of initial relative velocity of the maneuvering spacecraft with respect to the target spacecraft, each velocity component in the Hill's frame has been varied from 0 to 0.2 km/s and the results of the simulation have been obtained for different mission timings. Fig.9 shows the maximum error in control input as a function of the initial relative velocity while using the continuous thrust CW model. Further the results are also summarized in Table-2. It is deduced from the Table-2 that the maximum error in obtaining required control input increases as the relative velocity increases. However, the increment is low if we compare it for the low mission timing with the high mission timing, for example, if transfer takes place when the specified mission time is as low as $T=\left(\frac{\pi}{4n}\right)=694.2\,s$, there is only 1% error in the applied control input if the initial relative velocity of the maneuvering spacecraft is up to 0.346 km/s. Whereas when the mission time is increased to $T=\left(\frac{\pi}{n}\right)=2776.4\ s$, 1% error in applied control input exists if the initial relative velocity of the maneuvering spacecraft is as low as 0.06 km/s. Further, it is noticed that when the time of transfer has been increased to $T=\left(\frac{2\pi}{n}\right)=1388.4\ s$, there is a drastic increment in the control input error where is shoots as high as 6%, however, thereafter decreases and then again increases as the initial relative velocity of the maneuvering spacecraft increases.

Effect of the Target Spacecraft Orbit Altitude

Since, the analyses performed in subsections 5.1 and 5.2 deals with the target spacecraft in a low-Earth orbit with altitude of 400 km, the effect of changing the orbit altitude on the accuracy of the CW model is also studied.

Considering the initial state vector to be given as:

$$X\,(t=t_i) = [50\ \ 0.02\ \ 50\ \ 0.02\ \ 50\ \ 0.02\,]^T \tag{30}$$

The target spacecraft is located in an orbit of radius r_t. It is required to perform continuous low-thrust rendezvous so that, the final state vector becomes:

$$\boldsymbol{X}(t = t_i) = [\,0\ 0\ 0\ 0\ 0\ 0\,]^T \tag{31}$$

The time of the mission is set to be 3000 s. Optimization have been performed for different altitudes of the target spacecraft orbit, viz. LEO (altitude range 160-2000 km), MEO (altitude range 2000-35785 km), geosynchronous (35786 km) and HEO (altitude range 35786 km and above). Fig.10 shows the variation in the error in the control input as the altitude of the target orbit increases in LEO from 160 km to 2000 km. It can be noticed that the error is maximum at an altitude of 460 km and thereafter it starts decreasing. The trend continues for the MEO, shown in Fig.11. However, the slope of the error curve does decrease after 1700 km of target orbit altitude. In the case of HEO as shown in Fig.12 the control input error curve shows exponential decrement. An important observation that can be made here is that order of the error in control input while increasing the target spacecraft orbit altitude, is 10^{-3} for LEOs and MEOs, and 10^{-4} for HEOs, which is comparatively smaller if the errors with increase in initial relative distance and velocity are considered. The reason for the decrement in error with increase in altitude is because of the approximation made in the CW model that the relative separation between the target and the maneuvering spacecraft is relatively smaller as compared to the orbit altitude.

Conclusion

In this paper, the minimum propellant optimal relative transfer in the orbit using continuous thrust CW model has been studied. While comparing the results of optimization carried out using the approximate model developed by Clohessy and Wiltshire [1] with the non-linear model, it has been found that as the former model has limitations to its use due to approximations made in the derivation. Using CW model for continuous transfer beyond the limits of initial conditions can put extra burden on the guidance system and will result in extra propellant expenditure. Further, the various limits of relative distance and relative velocity under which the continuous thrust CW model can be used are estimated for a target orbit of radius 6778.14 km, which is also the approximate orbit of the International Space Station. Further, the trend in the control input error with the increase in the target orbit altitude has been studied and it has been found that due the approximation in the CW model, the error in control input decreases with the increase in the altitude of the target spacecraft orbit.

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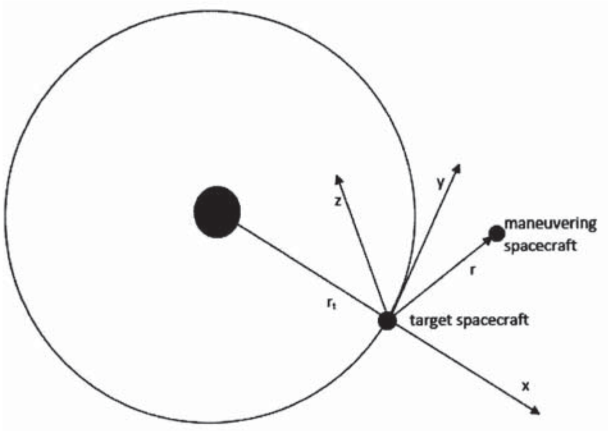

Fig.1 Hill's Frame and Configuration of Target and Maneuvering Spacecraft

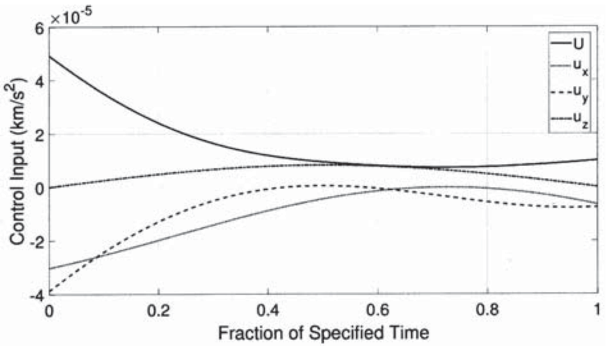

Fig.2 Trajectory of Control Input Obtained Using Continuous Thrust CW Model

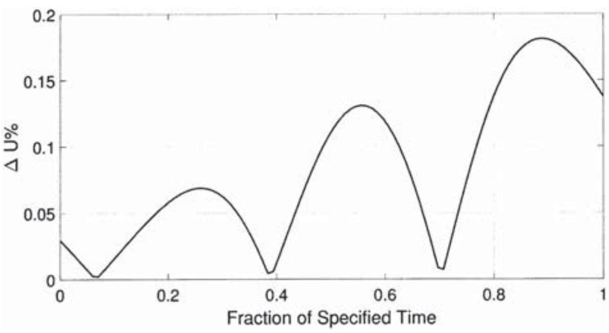

Fig.3 Error in Control Input

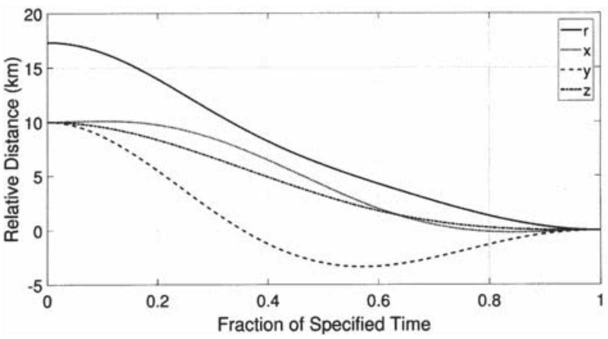

Fig.4 Trajectory of Relative Distance Obtained Using Continuous Thrust CW Model

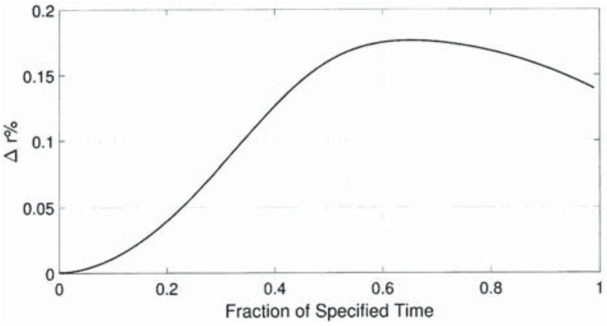

Fig.5 Error in Relative Distance

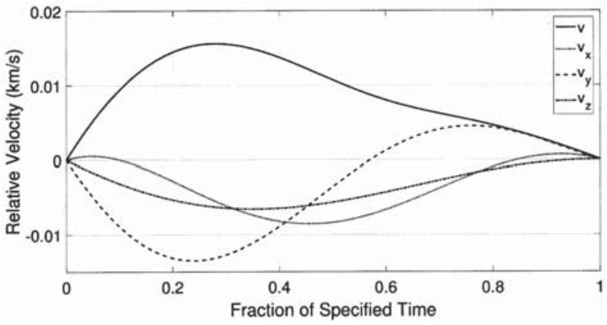

Fig.6 Trajectory of Relative Velocity Obtained Using Continuous Thrust CW Model

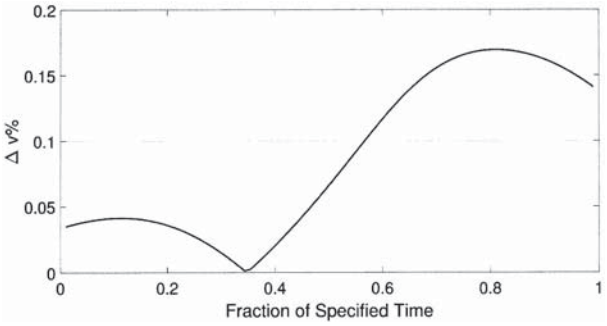

Fig.7 Error in Relative Velocity

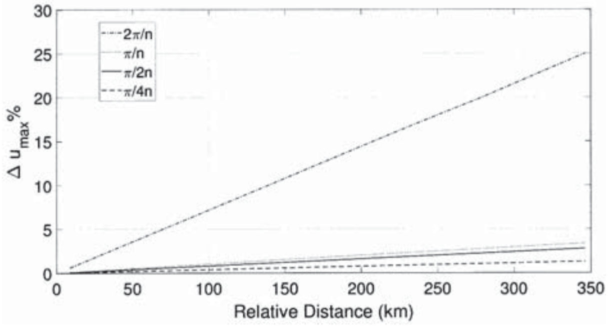

Fig.8 Error in Control Input with Increase in Initial Relative Distance

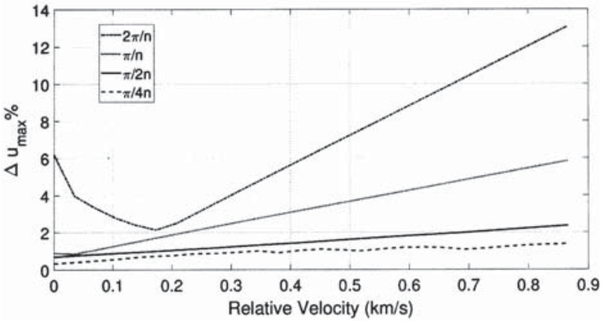

Fig.9 Error in Control Input with Increase in Initial Relative Velocity

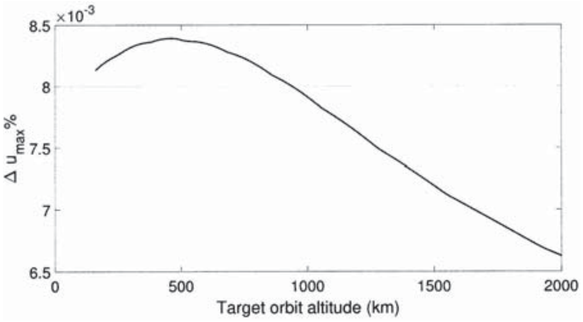

Fig.10 Error in Control Input with Increase in Orbit Altitude (LEO)

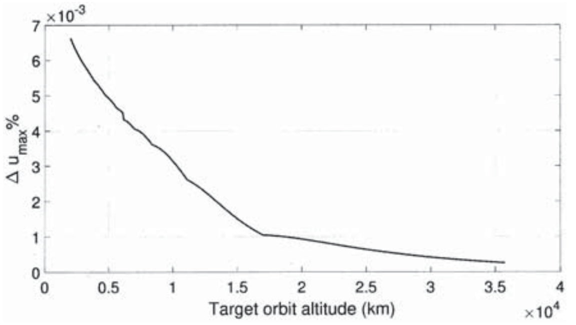

Fig.11 Error in Control Input with Increase in Orbit Altitude (MEO)

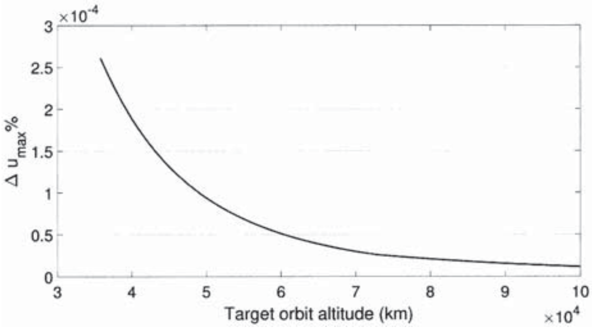

Fig.12 Error in Control Input with Increase in Orbit Altitude (HEO)